STATE OF STRESS COMPUTATION UTILIZING STRAIN GAGE INPUT AND A RESISTIVE COMPUTER

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June, 1969

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ABSTRACT

Determination of the state of stress at a point is possible by use of three or more values of normal strain acting on planes passing through the point. This determination is made by solving certain geometric and stress-strain relations called rosette equations. Solution of these equations involves a large amount of computation. This briefly reviews the rosette equations and various methods developed to simplify their solution. A resistive analog computer is developed which bypasses the equations themselves and produces, instead, the Mohr's Circle representation of the state of stress. Use of the computer with several types of input/output devices is described and the results of such operation is evaluated. The major conclusion is that solution of the rosette problem by means of a resistive analog computer is a practical method which yields adequate and easily obtainable results.

Thesis Supervisor: Dr. William M. Murray
Title: Professor of Mechanical Engineering

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SYMBOL LIST

Symbol	Meaning
B	Ballast resistor or dummy gage
E	Modulus of elasticity
H	Hydrostatic stress voltage terminal
k	Ratio of nondimensionalized stress circle radius to strain circle radius
N	Normal stress voltage terminal
R	Resistance
r	Specific resistance, radius
r_c	Corner specific resistance
r_{ch}	Modified corner specific resistance
r_g	Strain gage specific resistance
r_h	Hydrostatic specific resistance
r_m	Middle specific resistance
r_o	Outer specific resistance
r_p	Poisson's Ratio adjusting specific resistance
r_u	Net Poisson's Ratio adjusting specific resistance
r_q	Quadrant specific resistance
r_{qr}	Resultant quadrant specific resistance
$r_{1,2,3}$	Equivalent specific resistance 1, 2 and 3
r_ε	Strain circle radius
r_σ	Stress circle radius
R_o	Voltage divider resistance
R_s	Strain signal generator potentiometer
REF	Strain signal reference potential terminal
S	Shear stress voltage terminal

Symbol List (continued)

Symbol	Meaning
S_p	Power switch
$S_{1,2,3,4}$	Switches 1, 2, 3, and 4
V_b	Battery voltage
I, II, III, IV	Strain signal terminals I, II, III, and IV
γ	Shear strain
γ_{max}	Maximum shear strain
$\epsilon_{I,\ II,\ III,\ IV,\ a,\ b,\ c,\ d,\ x,\ y}$	Normal strain in the I, II, III, IV, a, b, c, d, x, or y direction
$\epsilon_{1,\ 2}$	Principal strains ($\epsilon_1 > \epsilon_2$)
ϵ_H	Hydrostatic strain component
ϕ	Angle measured counter-clockwise from reference axis to direction of algebraically larger principal stress
$\sigma_{x,y}$	Normal stress in the x, y direction
$\sigma_{1,2}$	Principal stresses ($\sigma_1 > \sigma_2$)
σ_H	Hydrostatic stress component
σ_{30}	Normal stress in the direction ϕ = 30 degrees
τ	Shear stress
τ_{max}	Maximum shear stress
μ	Poisson's Ratio

Section 1. Introduction

Strain gage rosettes are often used to provide information for the experimental determination of the state of stress at a point on the surface of a body. Analytic conversion of rosette data to state of stress solutions is a tedious, time-consuming job. Several types of computer have been developed to solve the rosette equation problem, but most have been rather complex, bulky, and expensive.

This report describes the development and use of a simple, resistive analog computer which solves the rosette problem by use of an analog of Mohr's Circle rather than by direct solution of the rosette equations. This device may be used as a data conversion link between various types of strain signal input devices and various types of stress indication output devices.

Representative examples of the computer's use are included in the report as well as recommendations for further improvement in its design and use.

Section 2. The Problem

The electrical strain signal derived from a single strain gage can be interpreted in terms of stress only in the event that the gage is mounted in a region of uniaxial stress and that the orientation of the gage with respect to the non-zero principal stress is known.

In general, three pieces of information are required in order to determine the state of stress at a point on the surface of a body. This information can take several forms. For example, the values of the two principal stresses in the surface and the direction of either

one with respect to an arbitrary reference line suffice to determine the state of stress. More generally, the values of the normal stresses in any three specified directions will yield the state of stress. Another general example of sufficient information is the value of the normal strains in any three specified directions. The latter case may arise when strain gage information is used and also requires knowledge of the values of the Modulus of Elasticity and Poisson's Ratio for the material being tested. In the general bi-axial stress case then, at least three strain gages are required in order to be able to determine the state of stress at a point.

The conversion of strain information to stress information is simplified if fixed, known, relative strain gage positions are used. The simplification arises from the fact that the necessary trigonometric functions become fixed values and standard reference directions can be chosen. Several convenient strain gage configurations, or rosettes, have been developed and marketed. The most common of these rosettes, illustrated in skeleton form in Figure 1, are the Delta Rosette and the Rectangular Rosette.

Useful variations of the Delta and Rectangular Rosettes are the Tee-Delta and the Four Gage Rectangular Rosettes, which are illustrated in Figure 2. The major advantage of the latter rosettes over their three-gage counterparts is that the fourth gage, which is redundant and theoretically unnecessary, provides a check on the experimental information provided by the other three gages.

The state of stress may be determined from recorded rosette strain values or, given proper equipment, directly from the rosette

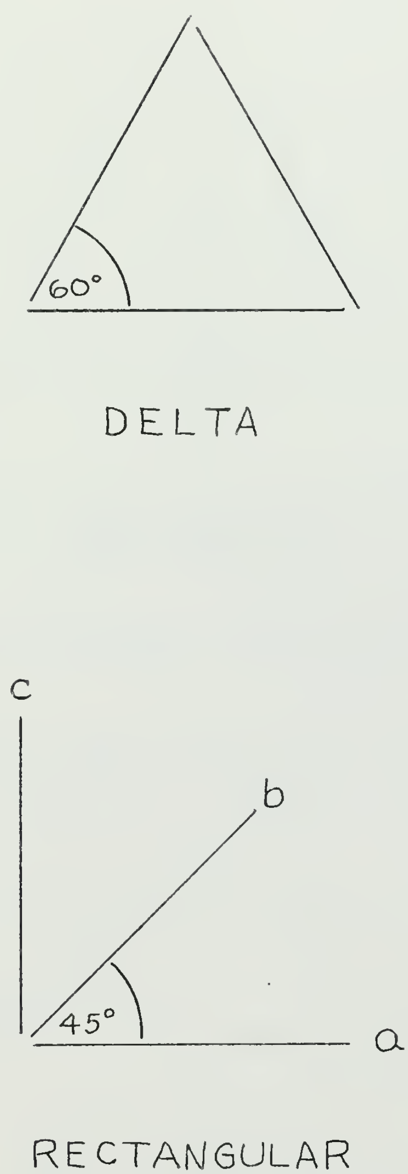
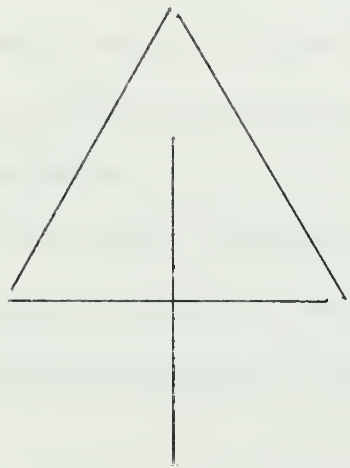
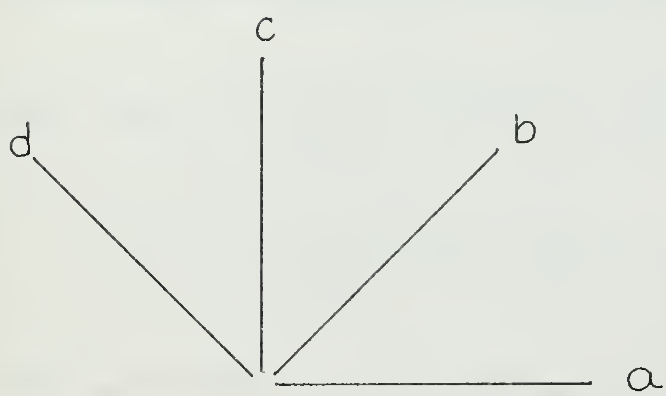


Figure 1 Three Element Rosettes



TEE–DELTA



RECTANGULAR

Figure 2 Four Element Rosettes

strain signals.

Section 3. The Rosette Equations

As mentioned in the previous section, the use of standard rosette configurations simplifies the equations relating the set of strain values to the corresponding state of stress. Still, the so-called rosette equations are not simple and involve a good deal of arithmetic manipulation in their solution.

The solution of the rosette equations in terms of stress yields the values of the principal stresses, σ_1 and σ_2, and the angle ϕ between a reference direction and the direction of the algebraically larger principal stress.

Development of the rosette equations is based on the following relations between the principal stresses and the principal strains, ε_1 and ε_2 for an isotropic, elastic medium subjected to conditions of plane stress:

$$(1) \qquad \sigma_1 = \frac{E}{1 - \mu^2} (\varepsilon_1 + \mu\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1 - \mu^2} (\mu\varepsilon_1 + \varepsilon_2).$$

Also used are the tensor transformation equations relating the principal strains to the normal and shear strains measured with respect to a given Cartesian (x and y) coordinate system:

$$(2) \qquad \varepsilon_1 = \frac{\varepsilon x + \varepsilon y}{2} + \frac{1}{2} \sqrt{(\varepsilon x - \varepsilon y)^2 + \alpha xy^2}$$

$$\varepsilon_2 = \frac{\varepsilon x + \varepsilon y}{2} - \frac{1}{2} \sqrt{(\varepsilon x - \varepsilon y)^2 + \alpha xy^2}$$

and

$$\tan 2\phi = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} .$$

where ϕ is the angle measured in the counter-clockwise direction from the x axis to the ε_1 axis.

Certain components of these equations arise quite often and are given special names. For example,

$$\varepsilon_H = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{\varepsilon_1 + \varepsilon_2}{2} =$$

the hydrostatic strain component and corresponds to the center of the Mohr's Circle representation of the state of strain tensor, and

$$\frac{\gamma\ max}{2} = \frac{\varepsilon_x - \varepsilon_y}{2 \cos 2\phi} = \frac{\varepsilon_1 - \varepsilon_2}{2} =$$

one half the maximum value of the shear strain and corresponds to the radius of the strain Mohr's Circle.

The definitions of the hydrostatic stress component (σ_H) and the maximum shear stress (τ_{max}) have forms similar to Equations 3 and 4, respectively. They are related, however, to the geometry of the Mohr's Circle representation of the state of stress tensor. ((1))*

As an example of rosette equation complexity when developed for a specific standard rosette configuration, one may consider the resulting equations for the Rectangular Rosette of Figure 1.

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*Numbers enclosed in brackets refer to references listed at the end of the report.

$$\sigma_1, \sigma_2 = \frac{E}{1-u} \cdot \frac{\varepsilon_a+\varepsilon_c}{2} \pm \frac{1}{2} \cdot \frac{E}{1+u} \sqrt{(\varepsilon_a-\varepsilon_c)^2 + [2\varepsilon_b-(\varepsilon_a+\varepsilon_b)]^2} \qquad (5)$$

where $+$ is used for σ_1, and

$$\tan 2\phi = \frac{2\varepsilon_b - (\varepsilon_a+\varepsilon_b)}{\varepsilon_a - \varepsilon_c} \qquad (6)$$

where ϕ is measured counter-clockwise from the a axis (Figure 1).

The solution of such equations, when much data is involved, is tedious at best, even with the use of hand calculators and printed forms. Digital computer solution can significantly reduce the drudgery and the incidence of arithmetic error, although data card preparation and computer availability can still cause the process to be rather long. Analog to digital conversion of the rosette strain signals reduces the data preparation time but drives up the cost of the process.

The advantage of an analog computer is that very little data manipulation is required before or during the problem solution.

Section 4. Review of Past Computer Methods

Various methods have been devised to reduce the computational effort required for rosette equation solution. They include grapho-analytic techniques ((2)), nomographs ((3)), and the use of electrical rosette computers ((4)), electronic rosette computers ((5)) and ((6)) mechanical rosette computers ((7)), and electro-mechanical rosette computers ((8)).

Graphical and nomographic methods alleviate some of the tedium involved in solution of the rosette problem. Unfortunately, they are time consuming and subject to such problems as scale error and interpolation error.

The earliest rosette computers were of the purely mechanical type which, in effect, perform the arithmetic functions of equations of the type of Equations 5 and 6. Electro-mechanical devices have also been developed which perform the same basic functions. These devices suffer from problems of mechanical backlash, mechanical complexity, and the fact that data must be introduced manually. Also, they are inherently bulky and heavy and rely on the use of highly paid skilled machinists in their manufacture.

Electrical and electronic computers have been devised which solve equations of the type of Equations 5 and 6 through use of the properties of sinusoidal alternating currents. Such devices can be operated in conjunction with the strain gage rosettes themselves. These devices have suffered from problems of circuit complexity and the requirement for large transformers and power supplies. The application of present day miniaturization techniques should enhance the design of such computers.

The following section discusses another type of computer which is easy to construct and use, compact, reasonably accurate, and which does not require an internal power supply.

Section 5. A Passive, Resistive State of Stress Computer

The major disadvantages of the rosette equation solution methods previously described, that is, excessive calculation, electrical, electronic and mechanical complexity, and high cost can be alleviated to a large extent by circumventing the rosette equations themselves.

Once the principle of the transformation of stress and strain tensors is understood, it is often more simple and meaningful to deal with the Mohr's Circle representations of the states of stress and strain

than with the transformation equations which they represent ((9)).

The strain values measured by the gages of a strain rosette are normal strains in the directions of the gage axis. These normal strain values can be represented by the projection of the corresponding points of the strain Mohr's Circle on the normal axis (abscissa). This is illustrated by the representation in Figure 3 of a Four Gage Rectangular Rosette and a representative strain Mohr's Circle.

The computer developed and described in this paper accepts normal strain information and uses it to solve the stress Mohr's Circle ((9)). This concept utilizes geometric relations between the Mohr's Circle for strain and the Mohr's Circle for stress and an analogy between electrical resistance and distance measured in Mohr Space, that is to say, distances measured in the Mohr's Circles. The major advantages yielded by this concept are that the resulting basic device is purely passive (does not need its own power supply) and that the electrical impedance of the device is purely resistive.

The remainder of this section will explain the development of the passive resistive computer in terms of Mohr Space and electrical circuit analysis. The detailed equations governing the actual design of the computer will be developed in the following section.

In order to relate the stress and strain Mohr's Circles in a simple geometric fashion, it is necessary to treat them both on a common scale. Since most commercial strain gate equipment is calibrated in terms of strain, the common scale used for this development is that of strain.

By adding Equations 1, one can show that

$$\sigma_H = \frac{\sigma_1 + \sigma_2}{2} = \frac{E}{1 - \mu^2} (1 + u) \frac{\varepsilon_1 + \varepsilon_2}{2} = \frac{E}{1-\mu} \varepsilon_H \qquad (7)$$

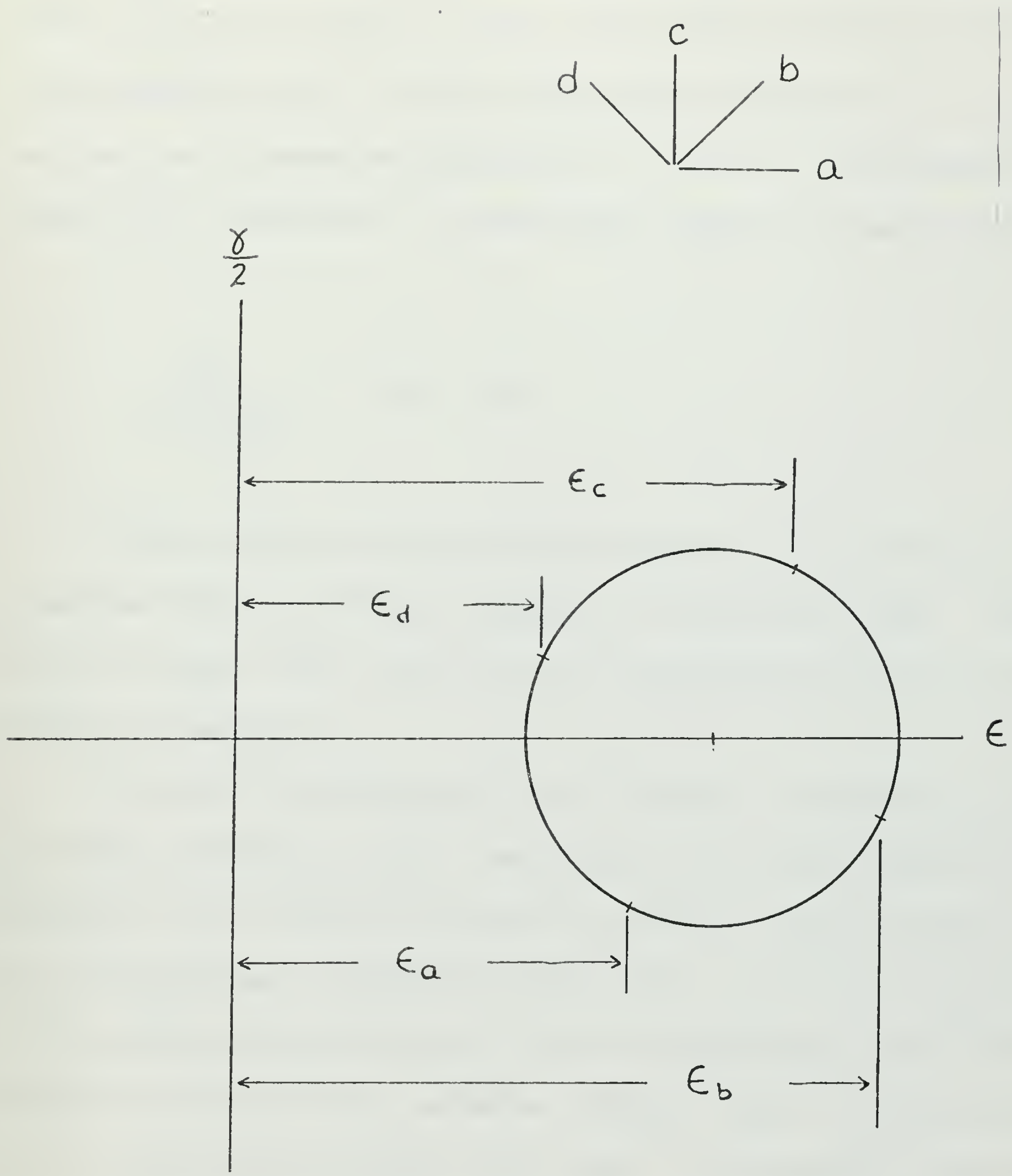

Figure 3 Strain Mohr's Circle

From this result it is apparent that stresses divided by the factor $\frac{E}{1 - \mu^2}$, have units of strain. The units of strain are themselves nondimensional; so these stresses may be referred to as nondimensionalized stresses and denoted by σ' and τ'. The reason for using $\frac{E}{1 - \mu^2}$, instead of E alone becomes clear if one notes that

$$\frac{\sigma_H}{\frac{E}{1 - \mu^2}} = \sigma'_H = \varepsilon'_H$$

That is, the hydrostatic strain component and the nondimensionalized hydrostatic stress component are identically equal. Therefore, the strain Mohr's Circle and the nondimensionalized stress Mohr's Circle are concentric.

The use of the concentric circles imposes a relationship between the radii of the two circles. The radius of the strain circle is proportional to the difference between the principal strains; which is, in turn, proportional to the maximum shear strain. Similarly, the radius of the nondimensionalized stress circle is proportional to the difference between the nondimensional principal streeses, which is also proportional to the nondimensional maximum shear stress. If Equations 1 are divided by the scale factor $\frac{E}{1 - \mu^2}$, and then subtracted, it is found that

$$\sigma_1 - \sigma_2 = \frac{1 - \mu}{1 + \mu} (\varepsilon_1 - \varepsilon_2). \quad *$$

*For the remainder of this paper the terms "stress" and "stress circle", and symbols relating to them, will refer to the nondimensionalized stress and stress circle unless otherwise noted.

Therefore, the ratio of the radius of the stress circle to that of the strain circle, which is equal to the ratio of the maximum shear stress to one half the maximum shear strain, is $\frac{1 - \mu}{1 + \mu}$.

Figure 4 depicts a typical strain circle with four points representing the strains corresponding to the four sequential directions of a Four Gage Rectangular Rosette. The gages yield direct information concerning only the normal strains in those directions. No direct information concerning the values of the associated shear strains is necessary for the solution of the state of stress problem. Also, the information developed by the strain gages is in the form of a small voltage which is directly proportional to the strain level. The strain circle (and the concentric stress circle) can therefore be plotted in units of voltage as well as in units of strain. Both types of units will be referred to in this paper; voltage will be implied when the circle in question is referred to as the resistance stress circle. It should be kept in mind, however, that while the shear strains may appear in terms of voltage in the electrical Mohr's circle representation, there are no voltages produced by the strain gages themselves which are related directly to the associated shear strains.

If secants are drawn between the adjacent points a, b, c, and d, as shown in Figure 4, distance between two points on any line is proportional to the distance along the normal axis between the projection of the two points upon the normal axis. Likewise, if a linearly disposed resistor is connected between two terminals whose voltages with respect to a common reference potential are proportional to the normal strains corresponding to the directions of the associated strain gages, then the voltage of any point along that resistor with respect

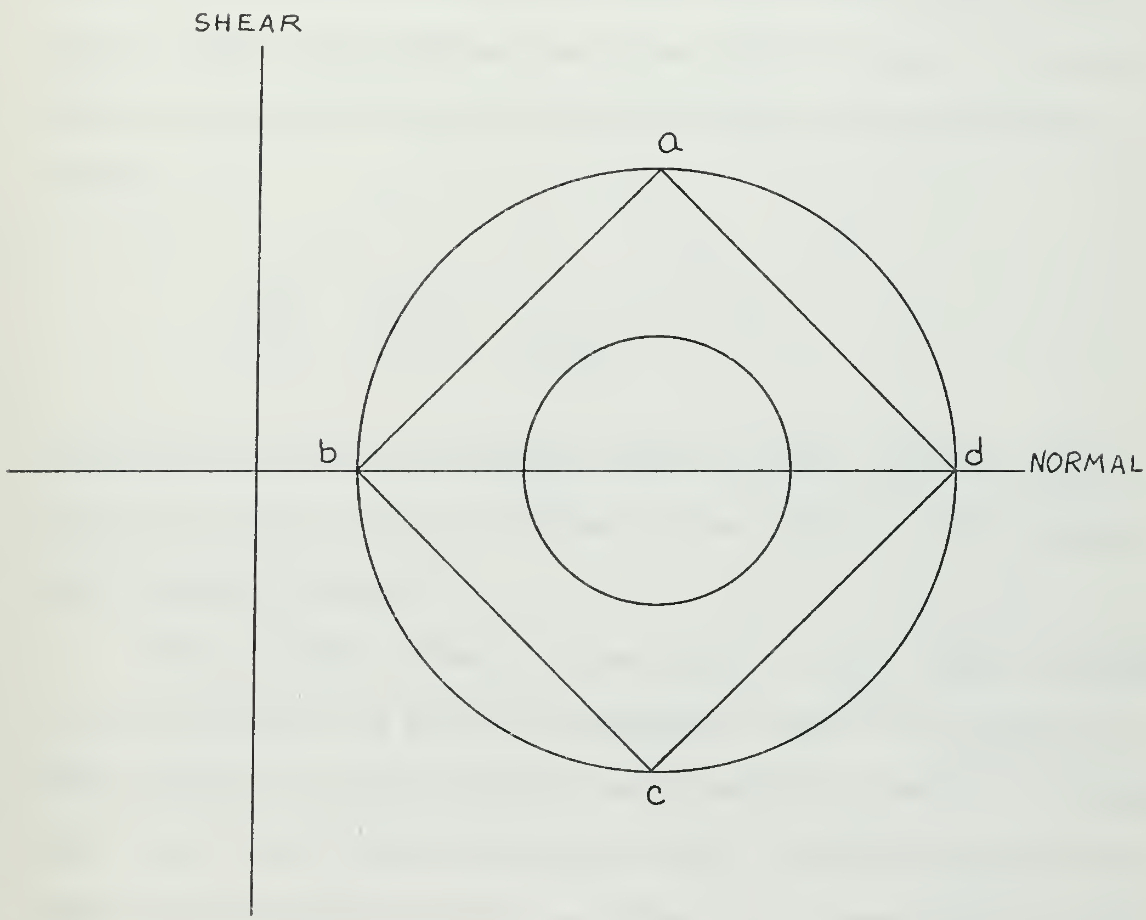

Figure 4 Mohr Space Figures

to the reference potential is proportional to the normal strain indicated by projection of the corresponding point of the Mohr Space secant upon the normal axis. This is merely a consequence of Ohm's Law.

Let us stop for the moment and look at the concentric stress circle. By virtue of the relation between the stress circle radius and the strain circle radius, the stress circle is seen to be smaller than the strain circle for all positive values of Poisson's Ratio. In fact, if

$$\frac{r_\sigma}{r_\varepsilon} = \frac{1 - \mu}{1 + \mu} < \frac{1}{\sqrt{2}} = 0.707 \quad ,$$

the stress circle can fit entirely inside a square inscribed within the strain circle, such as in Figure 4. The geometry of the limiting case is shown in Figure 5.

Thus, if four voltages representing the normal strain components in the directions of a Four Gage Rectangular Rosette are connected by four continuous, linearly disposed resistors in the form of a resistance square, then the voltage of any point along a resistor represents the normal strain component of the same point of the corresponding square inscribed in the Mohr's strain circle for that set of input strain values. Therefore, strain information is available in the form of voltage with respect to a reference potential for all points within the inscribed square.

The analog between electrical resistance and Mohr Space distance has developed to the point now where it becomes useful to think of the developing computer circuitry as being physically and electrically

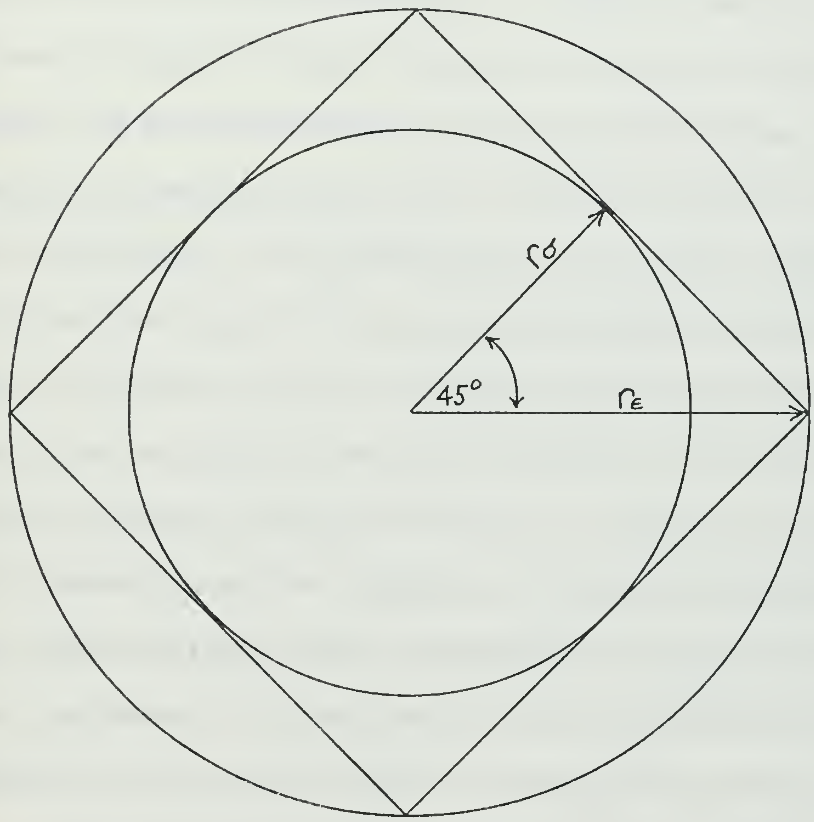

Figure 5 Limiting Ratio of Stress and Strain Circle Radii

laid out in the same form as the Mohr Space figures. Thus, terms such as "the resistance square" and "the resistance stress circle" will serve to keep the basis for the analogy in view even though the actual circuits need not be laid out in that manner.

If a linearly disposed, circular potentiometer winding, that is, a potentiometer winding whose incremental resistance is proportional to the arc angle traversed, is suitably connected to the resistors forming the square inscribed within the strain circle, voltages are developed along the potentiometer winding which, when measured with respect to the same reference potential used for the strain indications, are proportional to the normal stress corresponding to the position along the circular potentiometer winding. These voltages are picked up with a brush whose angular position is known. A second brush may also be used which contacts the circular potentiometer winding 90 degrees away from the first, or normal stress, brush. As will be shown in the next section, the voltage difference between this point and the voltage level representing the hydrostatic stress and strain components is proportional to the shear stress associated with the normal stress measured with the normal stress brush.

In order to electrically obtain an exact stress circle, the circular potentiometer winding (hereafter called the resistance stress circle) and the resistance square would have to be connected at an infinite number of points. Construction of a practical computer dictates that an approximation to the stress circle be made in order that only a finite number of connections be made between the resistance square and the resistance stress circle. The angular and magnitude

errors produced by this approximation depend on the polygon used to approximate the stress circle, but the size of the errors dwindles rapidly for regular polygons as the number of sides is increased.

The development of the resistance relationships for suitable connection between the resistance square and the resistance stress circle based upon a dodecagon approximation to the circle is presented in the following section. The use of a dodecagon, which is a twelve sided, equilateral polygon, results in a maximum error in stress magnitude of 3.4% and a maximum Mohr Space angular error of about 0.13 degree.

Section 6. Circuit Analysis

The purpose of the resistance networks of the computer described in this paper is to produce an analog representation of the (approximation to the) stress Mohr's Circle. The networks are designed such that the twelve apexes of the resistance dodecagon coincide with the correct values of the stress circle. The normal stress voltages produced at these twelve points are therefore the same as the voltages produced at the same points on a perfect resistance stress circle. The voltages developed at points along the dodecagon-connected circular potentiometer between the apexes vary linearly as the Mohr Space angle 2ϕ, and the stress circle will therefore be developed in the form of a dodecagon. This paper will continue to speak of the analog representation as the resistance stress circle; although it should be recognized that the actual representation is a dodecagon.

In order to develop the required relationships between the various resistances, the simple case of pure shear will be examined. Further, it is assumed that gages a and c (Figure 2) are aligned with

the algebraically larger and smaller principal strains, respectively.

The algebra involved in finding the relationships has been considerably simplified by extensive use of normalization. First of all, the circular potentiometer winding is divided into twelve equal portions, each representing 30 degrees of arc of the stress circle. All resistances are divided by the resistance of one section of the potentiometer winding measured before the winding is closed on itself to form the resistance circle. The resultant specific resistances are used in the calculations that follow. Likewise, all stress voltages and strain voltages are normalized by dividing by the voltage level corresponding to the maximum principal strain. The final simplification is to define a new constant k as the ratio between the stress circle radius and the strain circle radius. That is,

$$k = \frac{1 - \mu}{1 + \mu} \quad .$$

The normalized Mohr Space representation of pure shear and the circuit to be analyzed in terms of specific resistances are illustrated in Figures 6 and 7, respectively.

In the first quadrant of the Mohr Space, the points of agreement between the real stress circle and the dodecagon approximation to the stress circle lie in the directions $2\phi = 0$, 30, 60, and 90 degrees measured counter-clockwise from the Mohr Space direction coinciding with the axis of gage a. Thus, for pure shear measured with the indicated rosette orientation, the voltage levels on the resistance circle at the indicated angular positions must be k, $\frac{\sqrt{3}}{2}$ k, $k/2$, and 0, respectively.

Figure 7 shows that, because of symmetry, only four different specific resistances are necessary for the basic circuit, and one of

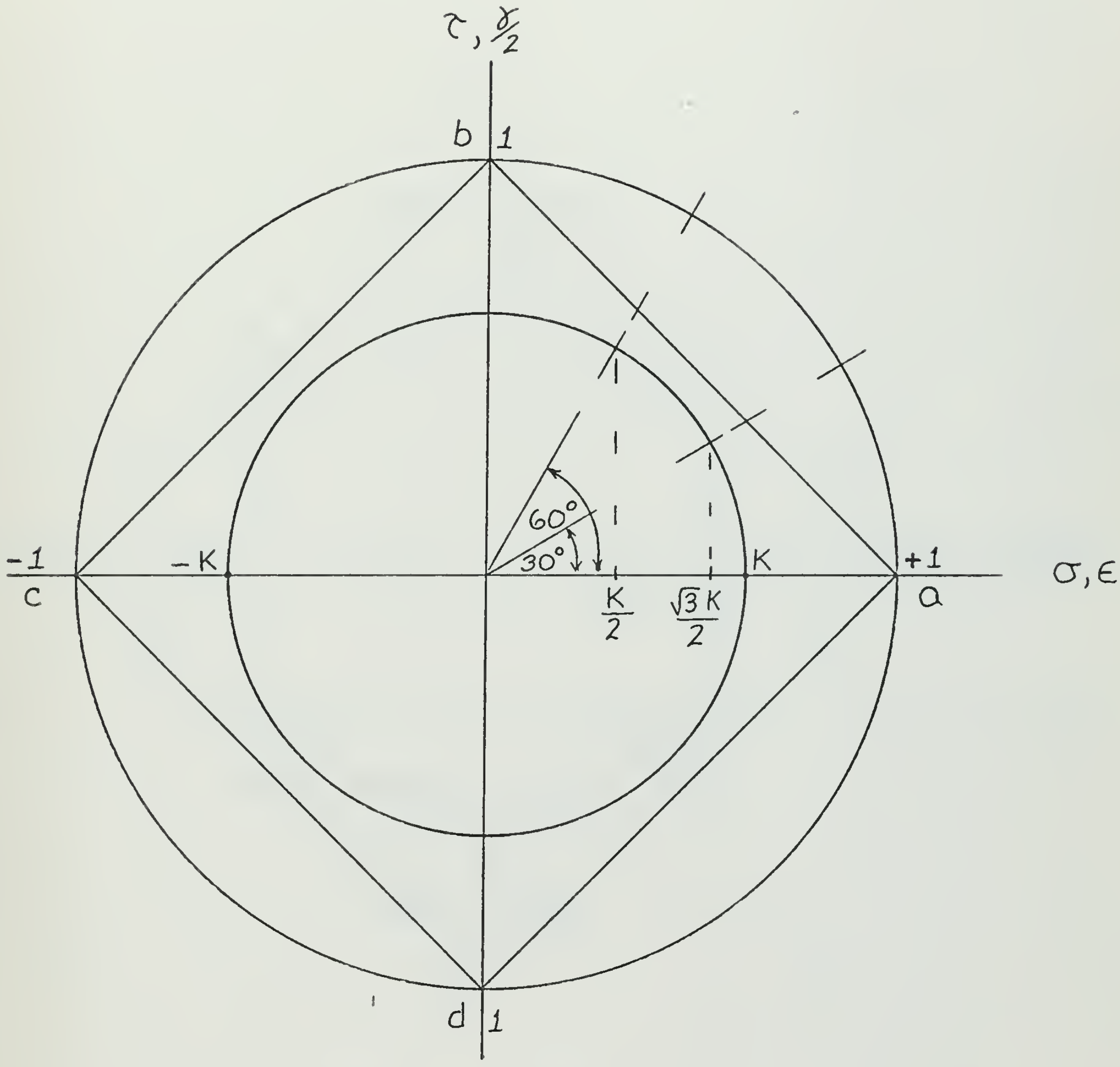

Figure 6 Pure Shear Stress and Strain Circles

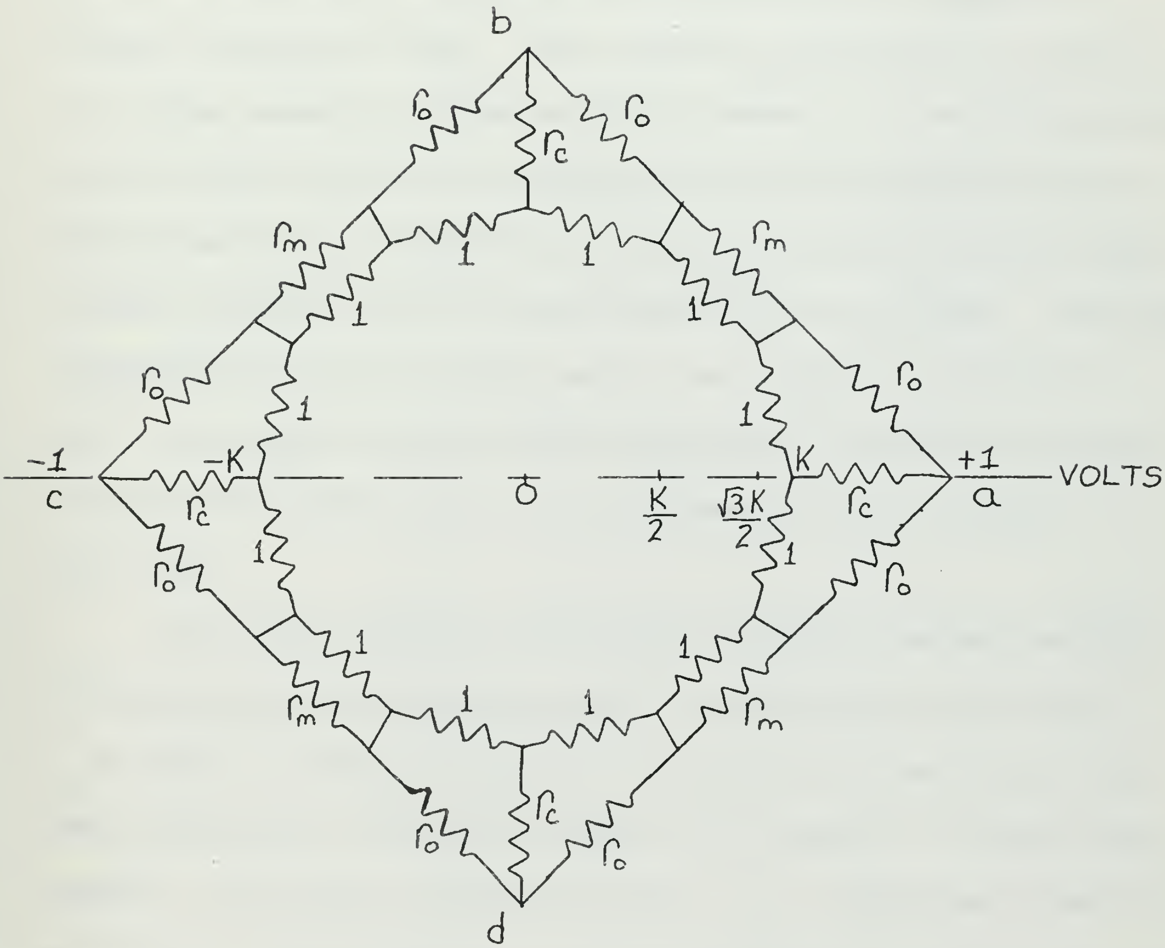

Figure 7 Computing Network with Pure Shear Voltages

these, the specific resistance of a resistance stress circle section, is unity. For the special case under investigation it is also evident that no voltage drop exists across the resistors r_c located at b and d because both the normal stress and the normal strain for those directions are zero. Therefore, no current flows through those two particular resistors and the entire circuit appears as a balanced Wheatstone bridge. This condition is maintained so long as the voltage measuring devices connected between the normal stress brush and the strain indication reference potential and between the shear stress brush and the hydrostatic stress potential have an effectively infinite impedance and do not draw current from the circuit. Additional simplification is now possible because the resistors r_c at points b and d, and only those two, may be eliminated, joining the resistance stress circle to the corners of the resistance square at points b and d only.

The first quadrant is now mentally removed from the remainder of the circuit and examined by itself. In doing this, the resistor r_c at point a must be replaced by a resistor of value $2r_c$ so that, when combined with a resistor of value $2r_c$ at point a in the fourth quadrant, the parallel resistance will be r_c. Another way of arriving at the same conclusion is to note that only half of the current flowing through the actual resistor r_c flows through the first quadrant; therefore, the resistance asociated with half the current must be $2r_c$ in order to produce the full voltage drop.

The first quadrant, modified to account for the zero voltage drop across one corner resistor and to account for the half current flow through the other corner resistor, is redrawn in Figure 8. This figure

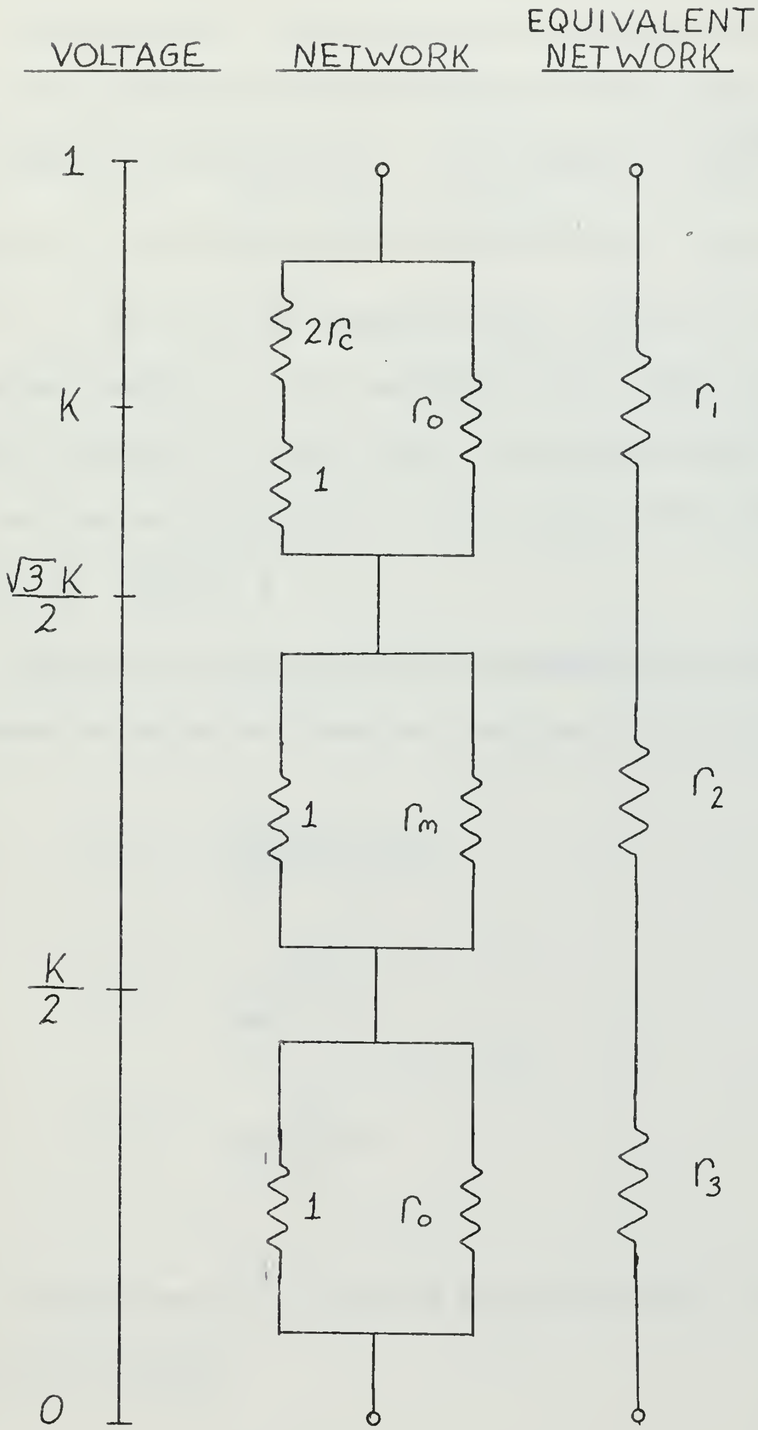

Figure 8 Equivalent Computing Network Resistances

also includes the required voltage levels at various junctures in the quadrant as well as an equivalent circuit of three series resistors.

The equivalent resistances are now considered to be analogs of the corresponding horizontal distances in Mohr Space. That is, r_1 is the analog of the distance $\varepsilon_1 - \sigma_{30}$, which is $\frac{2 - k\sqrt{3}}{2}$.

Similarly, r_2 is the analog of the distance $\sigma_{30} - \sigma_{60}$, which is $\frac{\sqrt{3} - 1}{2}$ k , and r_3 is the analog of $\sigma_{60} - \sigma_{90}$, or $k/2$. Also, within the network forming r_3, the resistance $2r_c$ is the analog of the difference between the strain circle and stress circle radii, which is $1 - k$, and the unit resistance is the analog of the distance $\sigma_1 - \sigma_{30}$, or $\frac{2 - \sqrt{3}}{2}$ k .

Using the relationships for determining the equivalent resistance of networks of series and parallel resistors, it is found that

$$r_1 = \frac{2r_c\ r_o + r_o}{2r_c + r_o + 1}$$

$$r_2 = \frac{r_m}{r_m + 1}$$

$$r_3 = \frac{r_o}{r_o + 1} .$$

Through use of the analogs described above, the following ratios can be formed.

$$\frac{2r_c}{1} = 2(2 + \sqrt{3}) \frac{(1 - k)}{k} \tag{9a}$$

$$\frac{r_1}{r_3} = \frac{2 - k\sqrt{3}}{k} = \frac{(2r_c + 1)(r_o + 1)}{(2r_c r_o + r_o + 1} \tag{9b}$$

$$\frac{r_1}{r_2} = \frac{2 - k\sqrt{3}}{(\sqrt{3} - 1)k} = \frac{(2r_c r_o + r_o)(r_m + 1)}{r_m(2r_c r_o + r_o + 1)} \quad . \tag{9c}$$

Solving Equations 9 simultaneously in terms of the three unknown resistances r_c, r_m, and r_o yields the resistance relationships required such that the voltages produced on the resistance stress circle and measured to the same scale as the strain imput voltages are equal to the corresponding nondimensionalized stresses. The resultant relationships are given by Equations 10.

$$r_c = (2 + \sqrt{3})\ (1 - k)/k \tag{10a}$$

$$r_m = (1 + \sqrt{3}) - (2 + \sqrt{3})k \tag{10b}$$

$$r_o = \frac{(1 + \sqrt{3}) - (2 + \sqrt{3})k}{k} \tag{10c}$$

At this level of development the computer could be built for one given value of Poisson's Ratio and would yield voltages with respect to the strain indication reference potential which are proportional to the normal stress in the direction indicated by the normal stress brush position. The information derived from this computer would include the maximum and minimum principal stress values (with use of the appropriate scaling factor) and their directions with respect

to, say, the axis of strain gage a. This information is sufficient to express the state of stress at the point where the rosette is mounted. A few simple modifications can be made to this basic computer in order to provide for different values of Poisson's Ratio and to allow the measurement of shear stress voltages with the second brush. These easily accomplished changes significantly increase the flexibility of the computer.

The size of the resistance stress circle relative to that of the resistance square is fixed by Equations 10 and by the value of k used in the design of the network. If, however, identical resistors are added between the corners of the resistance square and the strain gage terminals, that is, external to the computing network, the signals received by the computing network at the corners of the resistance square will be smaller in magnitude than the actual signals produced by the strain gage circuitry. The effect is the same as if the square were inscribed in an effective strain circle of smaller radius than the true strain circle. Since the inscribed square is smaller, the stress circle is likewise smaller, and the actual value of k is smaller as well. Thus, the resultant stress circle is based on a value of μ larger than that for which the computing network is designed. By properly choosing the value of the Poisson's Ratio adjusting resistance, r_p, any value of μ larger than the design value may be obtained up to the theoretical limit of 0.5.

The resistance r_q shown in Figure 9 is the resistance of one quadrant of the computing network and is the sum of the equivalent resistances r_1, r_2, and r_3. If μ is the value of Poisson's Ratio for the material to be tested, and μ_o is the value for which the

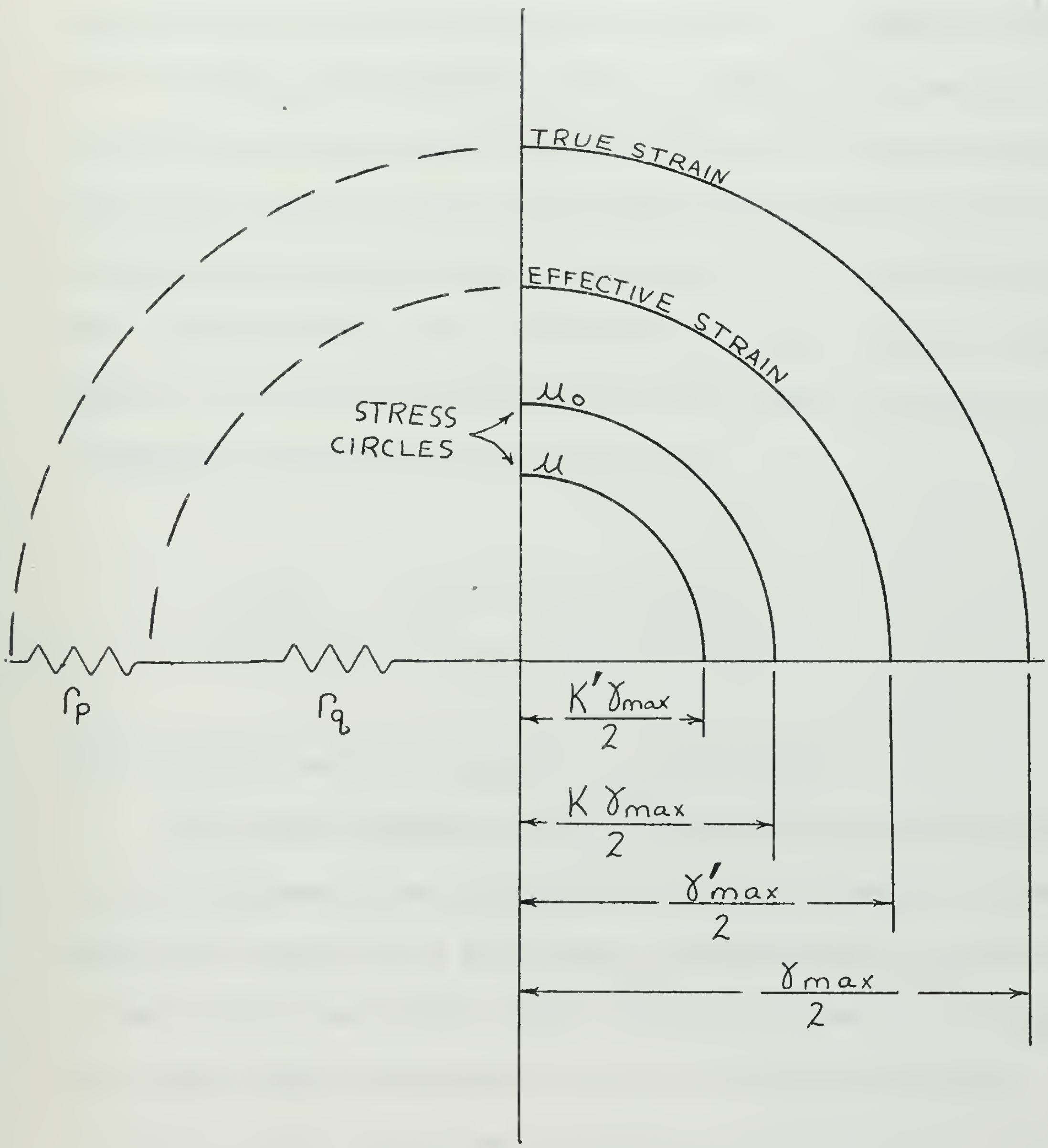

Figure 9 Poisson's Ratio Geometry Analog

computing network is designed, then the k' of Figure 9 is $\frac{1 - \mu}{1 + \mu}$ and k is $\frac{1 - \mu_o}{1 + \mu_o}$.

The radius of the true strain circle is $\frac{\gamma_{max}}{2}$ and the radius of the effective strain circle is $\frac{\gamma'_{max}}{2}$. The ratio of radii of the effective stress and strain circles must be k, since no change has been made to the computing network. Therefore, the ratio of radii of the strain circles must be equal to that of the stress circles, which means that the ratio of the radius of the effective strain circle to the radius of the true strain circle is k'/k. The analogs of these radii are seen to be, from Figure 9, r_q and $r_q + r_p$, respectively. Therefore, in order to obtain stress values based on the value μ with a computing network based on the value μ_o,

$$\frac{r_q}{r_q + r_p} = \frac{k'}{k} = \frac{(1 - \mu)\ (1 + \mu_o)}{(1 + \mu)\ (1 - \mu_o)} \tag{11}$$

from which the required value of r_p is determined.

The value determined for r_p includes the effective resistance of the strain gage network delivering the strain signal. For the usual Wheatstone Bridge type of strain gage excitation with a ballast resistor or dummy gage of resistance equal to that of the gage, the effective strain gage network resistance is one half the gage resistance r_g. The net Poisson's Ratio adjusting resistance r_μ is thus equal to $r_p - 1/2r_g$. Solving Equation 11 in terms of r_μ yields

$$r_\mu = r_q \frac{(1 + \mu)(1 - \mu_o)}{(1 - \mu)\ (1 + \mu_o)} - 1/2r_g \ . \tag{12}$$

The net Poisson's Ratio adjusting resistance can be added external to the computing network in the form of four ganged potentiometers calibrated in terms of μ. It appears to be more economical and simpler to adjust if the resistance values for the most common values of μ are calculated and then added in by means of a four section, ganged switch.

The modification which allows measurement of shear stress voltages involves production of a voltage which is proportional to the hydrostatic stress and the use of the second potentiometer brush mounted 90 degrees away from the normal stress brush.

From similar triangles, the distances $\overline{NX}$ and $\overline{AH}$ shown in Figure 10 are equal. The voltage XO is proportional to the normal stress σ_x in the x direction. The voltage AH, which is the voltage difference between the point A on the potentiometer winding and the hydrostatic stress voltage, is proportional to the shearing stress τ_{xy}. The hydrostatic stress voltage is the average of the voltages representing ε_a and ε_c, or the average of those representing ε_b and ε_d. This voltage can be developed by joining two opposite (diagonal) corners of the resistance square by two identical resistors r_h. The hydrostatic stress voltage is that voltage measured between the junction of the two resistors r_h and the strain indication reference potential. Although this voltage need be produced only from one pair of opposite corners, it is advantageous to use both pair and to develop two voltages. The voltages can then be compared during computer operation. If they are found to be different from each other, then there is some kind of incompatibility in the strain signals.

The value of r_h is not critical. The only constraint on its value comes from the fact that $2r_h$ is connected in parallel with the

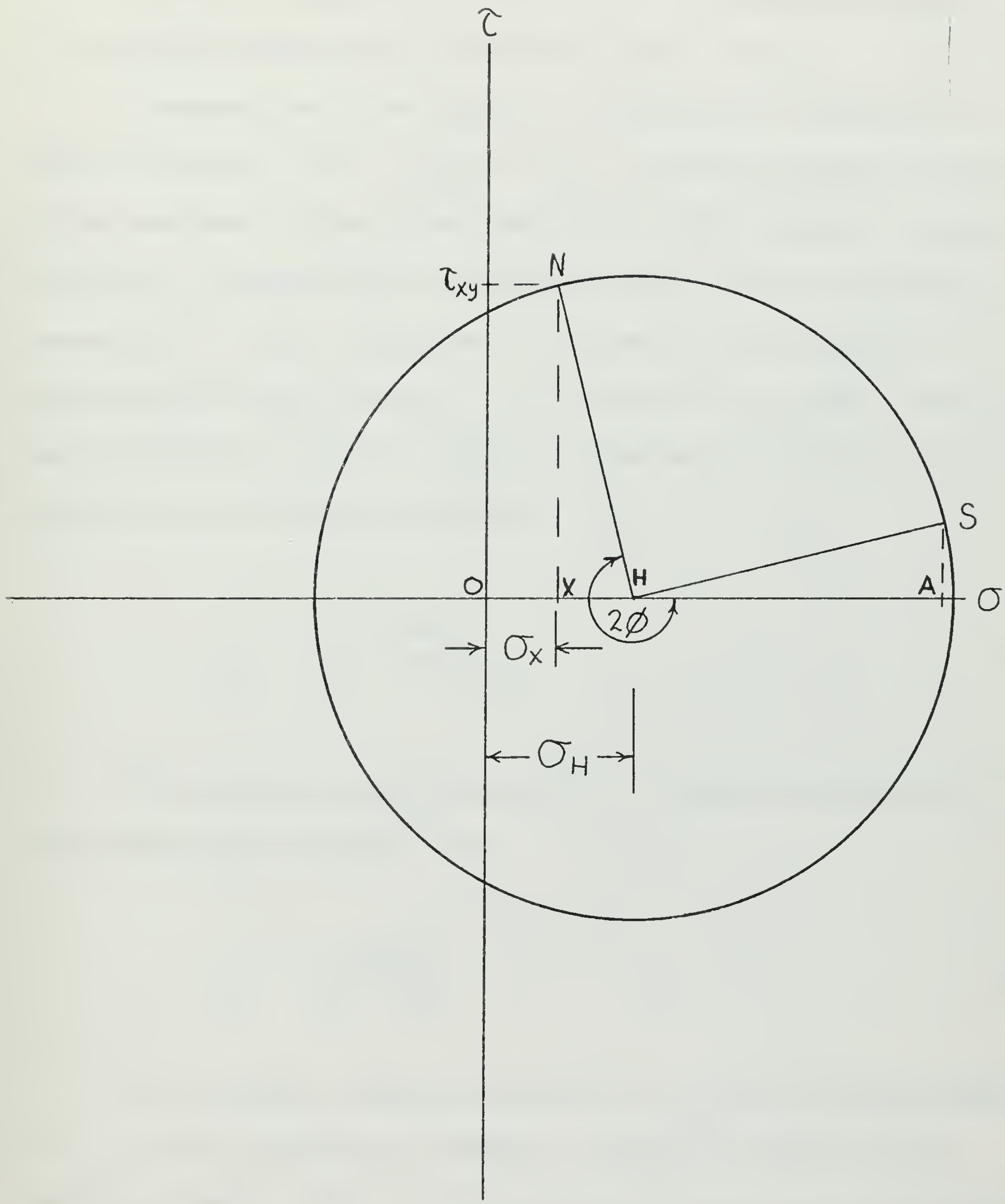

Figure 10 Shear Stress Measurement

two halves of the computing network, whose effective resistance is r_q. In order to minimize the effect of r_h on the impedance level of the entire circuit, $2r_h$ should be at least $10r_q$.

Construction of the computer is simplified by breaking r_h into two portions, $(1 - k)r_h$ and kr_h. The former component connects to the resistance square corner, while the latter component connects to the kr_h component from the opposite corner. The voltage drop across the $(1 - k)r_h$ component is the same as that across the corresponding corner resistor, r_c. There will be no effect, then, if the resistances r_c and $(1 - k)r_h$ are connected in parallel and then replaced by the equivalent resistance r_{ch}.

$$r_{ch} = \frac{(1 - k)r_c r_h}{r_c + (1 - k)r_h} \tag{13}$$

The presence of the resistance r_h reduces the quadrant resistance slightly to the value

$$r_{qr} = \frac{2r_q r_h}{r_q + 2r_h} \quad . \tag{14}$$

The resultant quadrant resistance, r_{qr}, must be used in place of r_q in any calculations involving resistances outside of the computing network, such as the calculations involving the net Poisson's Ratio adjusting resistance, r_μ .

The final computer design is illustrated in Figure 11 in terms of the specific resistances used in its development. The only additional calculation needed before actual construction is the conversion from specific resistances, r, to resistances, R.

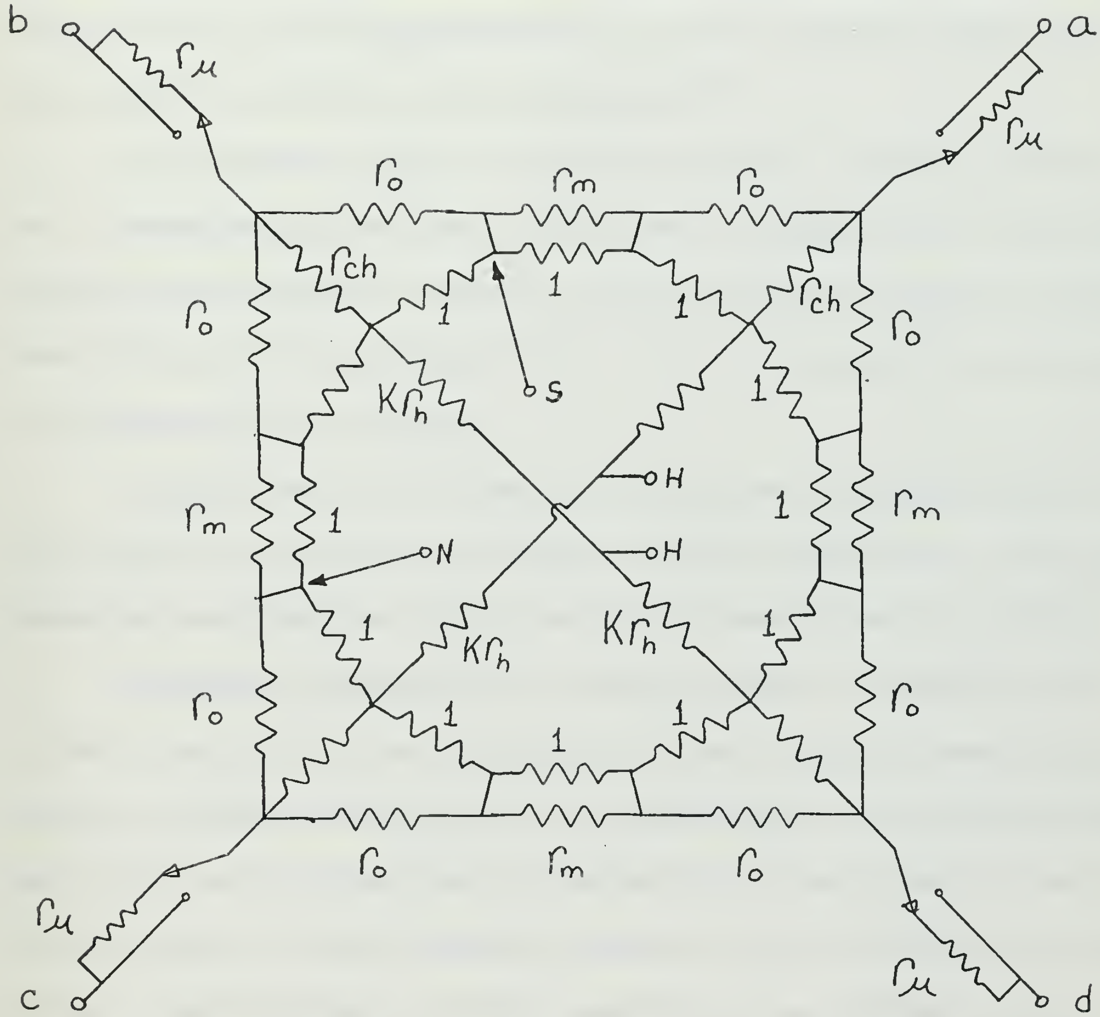

Figure 11 Computer Schematic Diagram

Section 7. Computer Construction and Use

The materials used to construct a prototype computer were, with one exception, inexpensive, off-the-shelf items. The single exception is the resistance ring potentiometer, which was a special order component and cost $100. The manufacturer estimated a per unit cost of $30. for quantities of twenty-five or more. The resistors used in the prototype were all 5% resistors; the use of 1% resistors would have increased the material costs an estimated $4.

Total material cost was approximately $117. for the prototype, and a breakdown of these costs is included in Appendix II. The author estimates that material costs for quantity production would be approximately $65. to $75., including items, such as a carrying case, which were not provided in the prototype.

Construction of the computer is simple and straightforward, and the design is well suited for printed circuit construction. The author therefore considers that labor costs for production would be minimal.

Prototype construction was centered around one terminal-type circuit board on which was mounted all of the computing network except for the resistance stress circle, or potentiometer. Multiple connection jacks were used for input/output terminals in order to match a variety of cable types. Input connections consist of four terminals for strain signal input (labelled I, II, III, and IV) and one terminal with no internal connections (labelled REF) which is used as a tie point for the strain indication reference potential. Output connections include REF, two terminals for the hydrostatic stress voltage (labelled H13 and H24, the numbers referring to the strain terminals from which the voltages are derived), one terminal for the normal stress signal (labelled N), and one terminal for the shear stress signal (labelled S).

One revolution of the potentiometer shaft represents 360 degrees in Mohr Space, but only 180 degrees in physical space. The shaft position is calibrated from 0 to 180 degrees measured counter-clockwise from the axis of the strain gage whose strain signal is fed to terminal I.

The prototype computer was based on a design value of $\mu = 0.28$. Although a twelve position, four section switch was installed to control Poisson's Ratio adjusting resistors, only one set of resistors was actually installed. This set produces a value of $\mu = 0.33$. The prototype is therefore suitable for use with steel and aluminum.

The calculated values of the computer resistances, as well as the values of the resistors actually installed, are listed along with the cost information in Appendix II.

The computer may be used with either AC or DC strain signals and with a variety of input and output devices. Although these external devices do not form part of the computer, their use is essential to the problem solution, and some combinations will be briefly discussed in the remainder of this section.

Regardless of the type of strain signal used (AC or DC) and regardless of the external devices used with the computer, there are two basic modes of computer operation. These are (a) the data reduction mode and (b) the active input mode.

In the data reduction mode of operation, previously recorded rosette strain data is fed into the computer input terminals in the form of a reference voltage and simulated strain signals, and stress voltages are produced.

In the active input mode of operation, strain signals and the strain signal reference potential are fed to the computer directly from

the strain gage circuitry as the test is performed, and stress voltages are produced.

The data reduction mode will be illustrated through one type of input device and several methods of output. The input device is basically a reversible polarity voltage divider with four linear potentiometers calibrated in strain units, such as micro-inches per inch. Circuit details and reference point identification are shown in Figure 12.

The signs of the strains are chosen by switches 1 through 4 and their magnitudes are set by use of the four calibrated potentiometers. The simulated strain signal terminals I-IV are connected to computer input terminals I-IV. The strain signal reference potential is chosen to be ground for this device in order to be able to develop positive and negative strain signals.

Once the strain levels have been set, proper connections made to the computer, and the power switch closed, one of the following methods may be used to obtain the output information:

1. A high impedance voltmeter is connected between appropriate terminals to measure output voltages.

   Any voltage found to exist between the two H terminals indicates an error in the strain signals introduced; that is, a non-zero voltage indicates that $\varepsilon_{I} + \varepsilon_{III} \neq \varepsilon_{II} + \varepsilon_{IV}$.

   The voltage measured between terminals N and REF (when multiplied by the proper scale factor) yields the value of the normal stress in the direction indicated by the position of the potentiometer shaft relative to the position corresponding to the axis position of the gage connected to terminal I. As the shaft is rotated, the maximum and minimum normal stresses and their

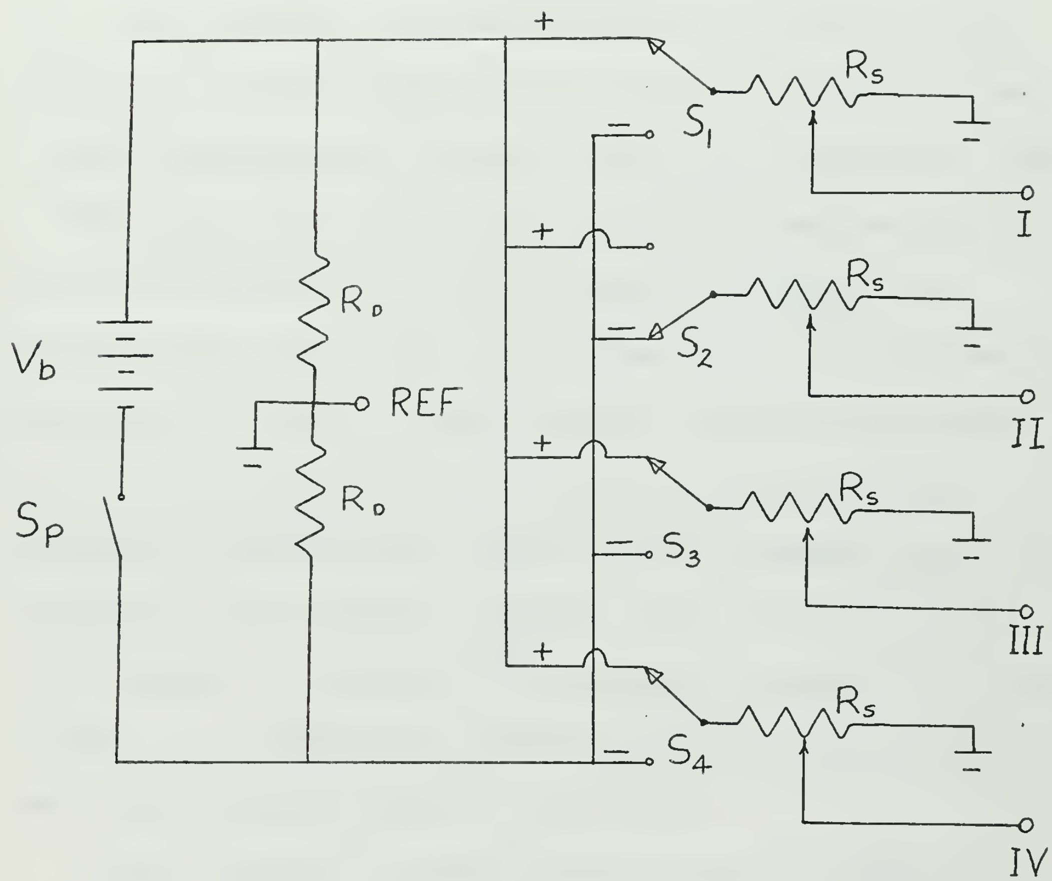

Figure 12 Strain Signal Generator

directions are recorded. This information is sufficient to completely specify the state of stress.

Although the information is not necessary for determination of the state of stress, the value of shear stress as a function of angular position may be found by measuring the voltage between terminals S and H.

2. An X-Y recorder or an oscilloscope will produce an actual display of the stress Mohr's Circle approximation if the normal stress voltage (between terminals N and REF) is introduced to the X axis input and the shear stress voltage (between terminals S and H) is introduced to the Y axis input. Rotation of the potentiometer shaft causes the recorder pen or the oscilloscope light spot to trace the stress circle. By marking the position of the pen or spot as the shaft passes through zero degrees (alignment with the axis of gage I) one can determine the direction of the principal stresses. The origin of the Mohr Space display is indicated by the no-signal position of the pen or spot. The appropriate scale factor must be applied to determine the stress magnitudes in stress units.

The X-Y recorder provides permanent graphic records of the state of stress, and permanent records can be obtained from the oscilloscope display by photographic means.

The active input mode is illustrated by the following examples:

1. A standard commercial strain indicator of the null-balance, reference bridge type is connected in half-bridge configuration as shown in Figure 13. The strain indicator is used both as

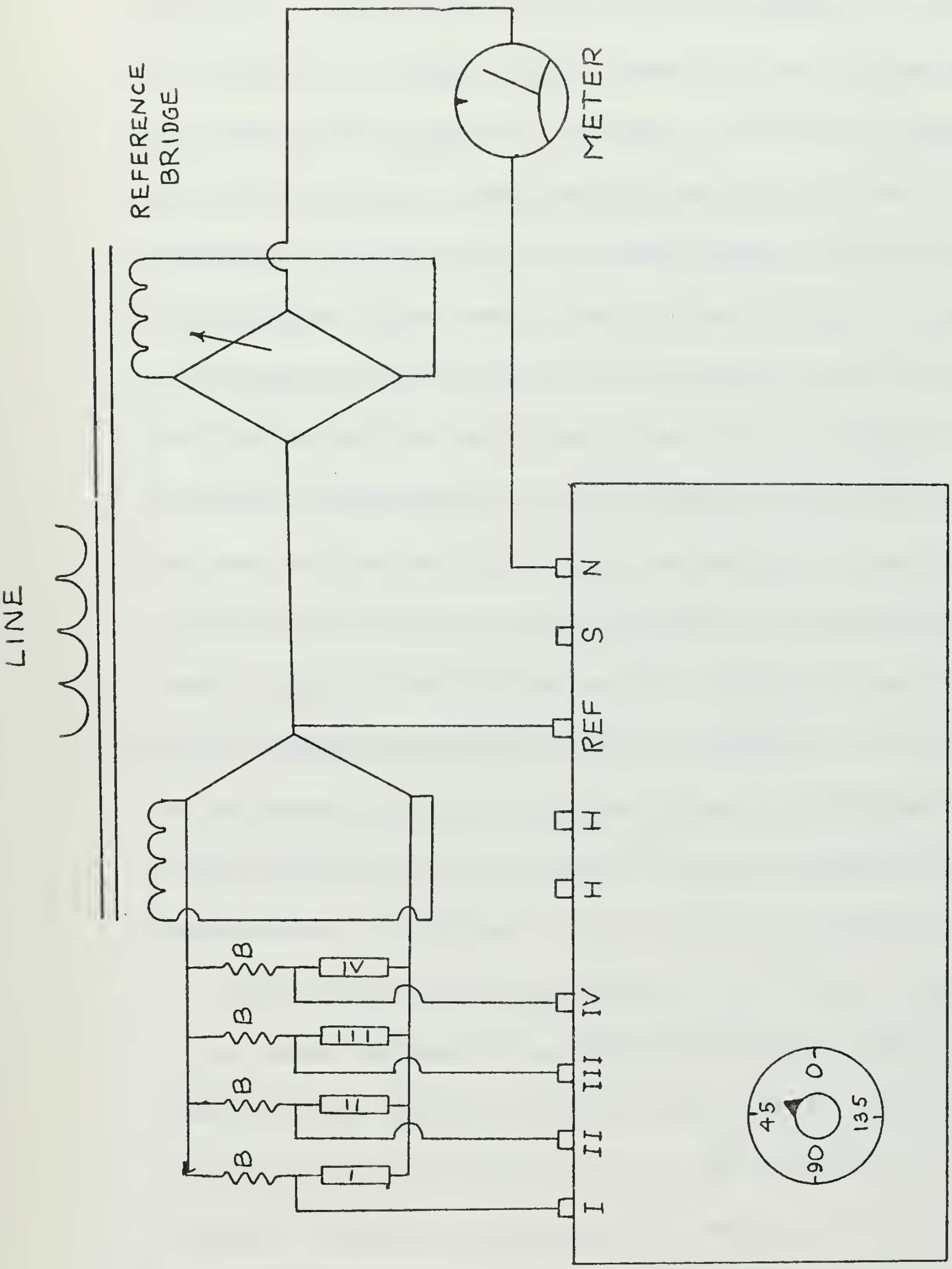

Figure 13 Strain Indicator Method

the strain gage circuit excitation source and as the output measuring device.

The ballast resistors (or dummy gages) marked B in Figure 13, are trimmed under no load conditions so as to give no variation of output signal (normal stress indication) as the potentiometer shaft is rotated. If this step cannot be accomplished, the no load readings should be recorded as a function of shaft position and then subtracted from the readings obtained under load. Readings are obtained by unbalancing the reference bridge such that the indicator meter is nulled and then noting the indicated stress value. It will be assumed from this point that the ballast resistors have been trimmed for zero no load variation, that the meter has been nulled for no load, and that the no load reading has been recorded. When load is applied, the voltage at the junction of each strain gage and its associated ballast resistor developed with respect to the reference potential is proportional to the normal strain experienced by the active gage. The four strain voltages so developed are introduced to the computer at terminals I through IV. The voltage between terminals N and REF is proportional to the normal stress in the direction corresponding to potentiometer shaft position and causes the strain indicator meter to deflect from the null position. The direction of the principal stresses is determined by rotating the shaft through 360 degrees and noting its position at the extreme deflections of the meter. The principal stress magnitudes (in terms of strain units) are determined by placing the shaft in first one

and then the other principal direction, unbalancing the reference bridge each time so as to null the meter, recording the indicated stress value, and then subtracting the no load indicated value. A voltage correction factor must be applied to these observed stress indications since the indicator dial calibration is based on the excitation voltage produced with one ballast resistor and active gage connected. The increased current drain produced by four such sets in parallel may cause the excitation voltage to fall somewhat. Since the strain signal is directly proportional to the magnitude of the excitation voltage, the true stress indication is equal to the observed stress indication multiplied by the ratio of the normal excitation voltage to excitation voltage produced with all four active gages connected simultaneously. This voltage ratio may be included in the scale factor used to convert the stress values in strain units to the stress values in stress units.

The magnitudes and directions of the principal stresses obtained by this method are sufficient to completely specify the state of stress.

2. In this example the strain gage circuit is excited by a DC source, and the reference potential is produced by voltage divider as shown in Figure 14. The output device chosen is an oscilloscope.

The ballast resistors must be trimmed at no load so as to produce no output voltage. With the normal stress voltage terminals (N and REF) connected to the oscilloscope X axis input and with the shear stress voltage terminals (S and H) connected to

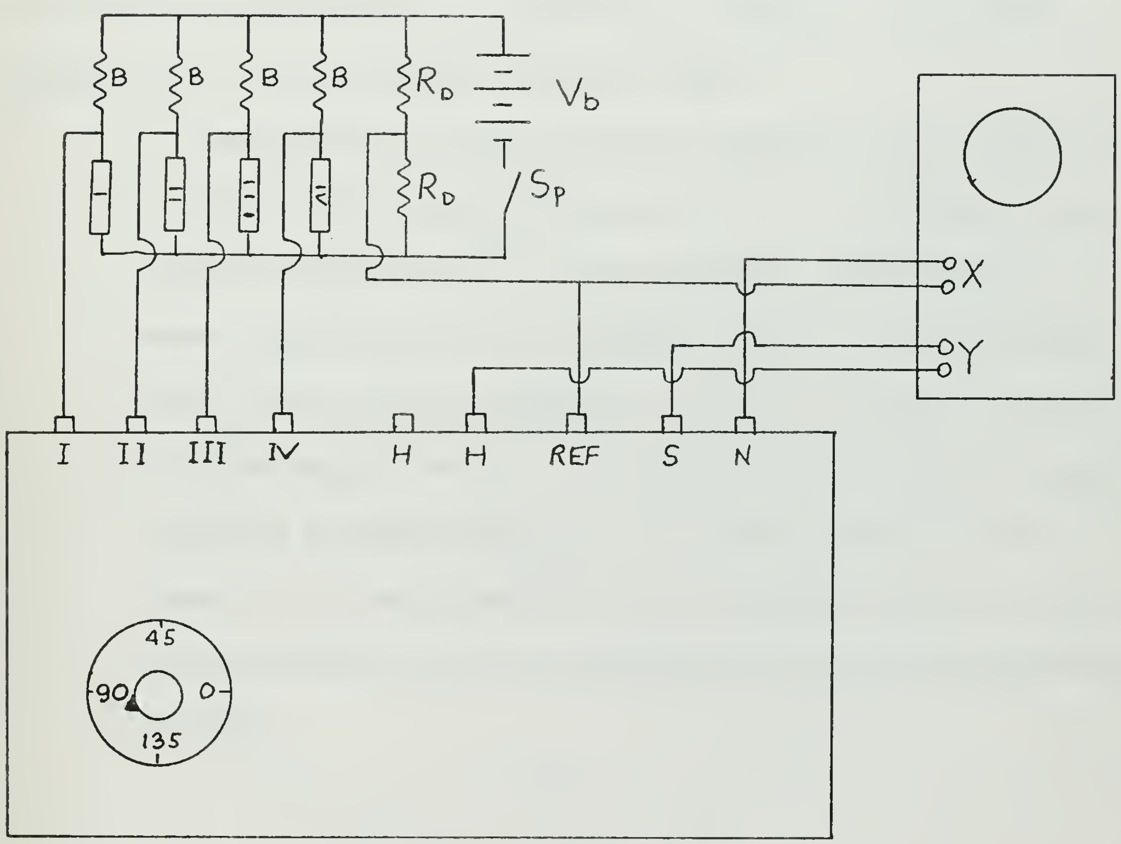

Figure 14 DC Excitation with Oscilloscope Output Display

the Y axis input, the oscilloscope light spot will trace the stress Mohr's Circle as the potentiometer shaft is rotated. The Mohr Space origin is indicated by the position of the spot at no load.

Section 8. Discussion of Results

Several methods described in the preceding section were used to evaluate the performance of the prototype computer. The results obtained with each method are discussed below.

1. Several sets of strain data were applied to the computer in the form of DC voltages of magnitude up to 4.5 volts. Voltage difference between the two H terminals was never found to exceed a few millivolts, indicating that the simulated strain signals were mutually compatible. An X-Y recorder was used as an output display device, and each set of strain data was used to produce a stress circle for $\mu = 0.28$ and for $\mu = 0.33$. Figure 15 is a reproduction of the recording of the two stress circles produced for the following set of strain data (measured in volts):

$$\begin{aligned} \varepsilon_{I} &= 0.0 \\ \varepsilon_{II} &= -1.5 \\ \varepsilon_{III} &= -3.0 \\ \varepsilon_{IV} &= -1.5 \end{aligned}$$

The larger stress circle is for $\mu = 0.28$. The dodecagon nature of the stress circle approximation is readily apparent. The dashed circle is the theoretical result for $\mu = 0.28$ and is seen to correspond closely to the computer results.

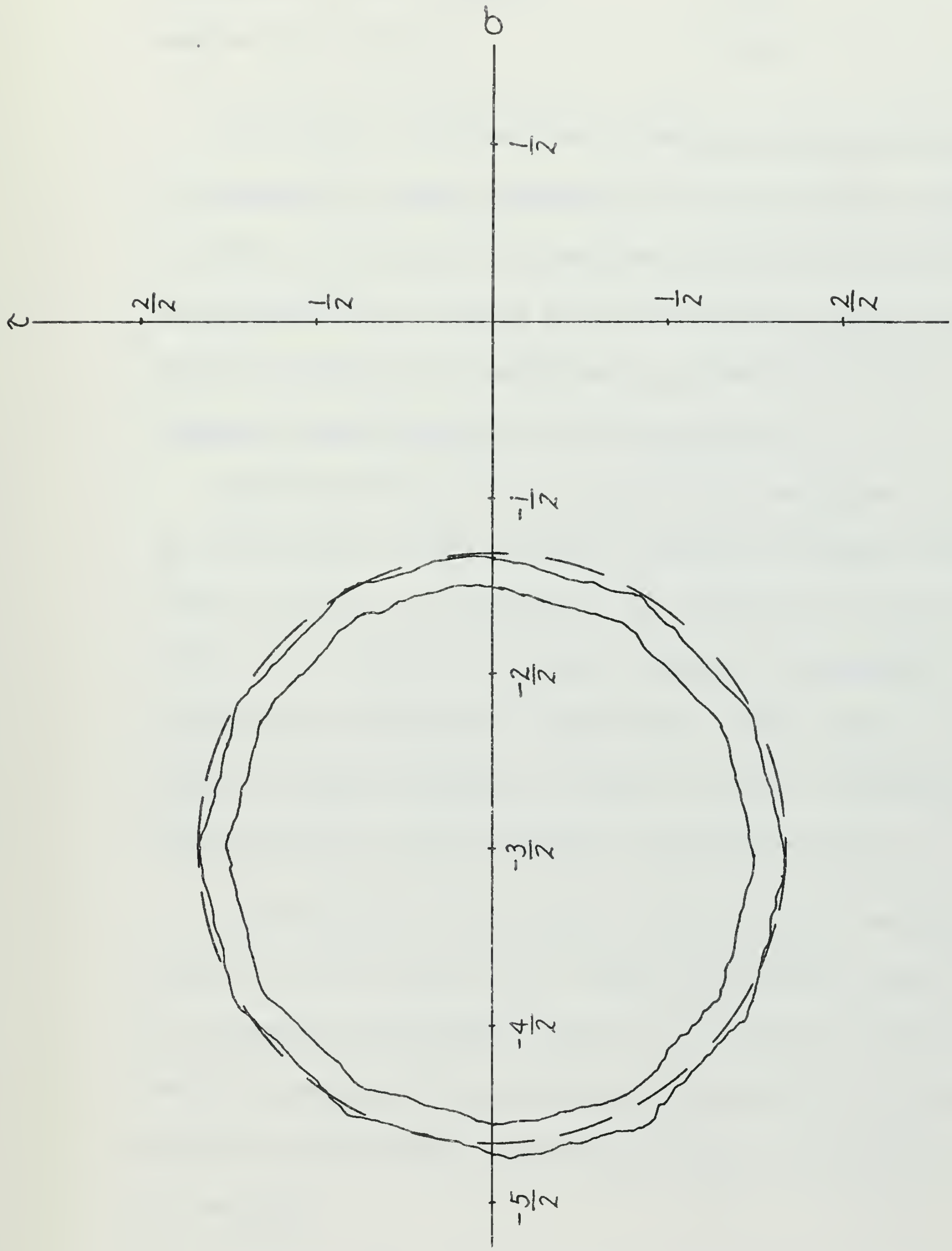

Figure 15 X-Y Recorder Representation of Stress Cycles

The maximum errors found in the cases studied are:

Hydrostatic stress component	2.4%
Maximum shear stress	4.4%

Care must be exercised when speaking of percent error in the hydrostatic stress component if the hydrostatic component is close to zero; that is, near the condition of pure shear. The maximum error found is only one percentage point greater than the maximum error predicted on geometric grounds for a dodecagon approximation to the stress circle.

Considering that the tolerance of the resistors used is 5%, the performance of the prototype computer with high level DC voltage signals is considered by the author to be quite satisfactory. The degree of accuracy attained is adequate for most engineering applications. Improvement in the degree of accuracy should be easily attainable through the use of precision resistors made to the calculated resistance values.

2. The simulated strain signals mentioned above were also applied to the computer while using oscilloscope output display. No permanent photographic records of the stress circle representations were made, but measurements made with the oscilloscope reticle yielded results comparable to those obtained with the X-Y recorder.

3. A number of AC simulated strain signals at rms levels of several volts and at various frequencies between 60 and 10,000 hertz were also investigated. Satisfactory stress circle representations were obtained with oscilloscope output display. The

X-Y recorder representations were less satisfactory; the information displayed yields the magnitude of the maximum shear stress and the direction of the principal stresses, but not the sign of the principal stresses. The reason for this ambiguity is that the X-Y recorder had no phase sensitive demodulator and therefore could not distinguish between positive and negative AC voltages. The X-Y recorder problem is manifested by the tracing of all four quadrants of the stress circle in the positive quadrant. The use of a recorder which incorporates a phase sensitive demodulator would overcome this difficulty.

4. The use of a Baldwin SR-4 strain indicator in the active input mode circuit of Figure 13 was also evaluated. Strain signals were obtained from a Four Gage Rectangular Rosette mounted on an aluminum cantilever beam loaded in bending. Dummy gages mounted on bendable sections of hacksaw blade were used in place of adjustable ballast resistors. The degree of bending of the individual blade sections was adjusted in an attempt to provide no load balancing. Limitations of the simple bending arrangement prevented precise balancing with the dummy gages and no load indications had to be recorded as a function of potentiometer shaft position. The principal directions of stress were found to correspond to the longitudinal and transverse axes of the beam, as predicted by theory. Stress values in units of micro-inches per inch were determined by theory. Stress values in units of micro-inches per inch were determined for the principal directions at five values of load, the results were plotted, and the slopes of the resultant

straight lines were recorded in terms of stress per pound of applied load. These results were then corrected by the ratio of the normal excitation voltage to that produced with the four sets of gages in simultaneous operation, which was found to be 1.21. The resultant principal stresses and the derived hydrostatic stress and maximum shear stress were then compared to the results predicted by simple beam theory as follows:

	Beam Theory	Computer Results	(%)
σ_1	48.4	48.4	0.0
σ_2	0.0	0.2	---
σ_H	24.2	24.3	0.4
τ_{max}	24.2	24.1	-0.4

The results obtained by this method for the cantilever beam in bending are extremely good.

5. The same beam problem and gage configuration described in 4 above were used with oscilloscope output display and with both DC excitation (Figure 14) and AC excitation provided by an audio frequency generator. No valid results were obtained with either form of excitation. Although a reproducible display was obtained, it bore no relation either to the state of stress or to a circle of any kind. The source of the spurious signal was eventually found to be electromagnetic noise in the laboratory environs.

Noise voltages induced in circuit leads were orders of magnitude larger than the output voltages to be measured. The author considers that adequate shielding of the computer and leads should solve this problem.

Section 9. Conclusions and Recommendations

The major conclusion of this report is that solution of the strain gage rosette state of stress problem by means of a Mohr's Circle resistance analog is practical. A passive, resistive analog computer of simple design has demonstrated the use of this concept with adequate accuracy. The prototype computer was found to be very flexible, in that it may be used in conjunction with several types of input and output devices.

Some conclusions concerning operation of the computer have also been reached.

Initial unbalance of four sets of gages in the active input mode, which results in apparent stress values under no load conditions, does not affect the results obtained. The additional work and time required to obtain and record initial no load readings as a function of shaft position detract from the usefulness of the computer. Initial balancing considerably simplifies the problem solution.

Care must be taken to include in the scale factor applied to strain indicator stress values a voltage correction factor to account for excitation voltage droop under the excess load of four sets of strain gages. This factor can be determined by measuring excitation voltages, or it can be included in an overall scale factor experimentally determined by use of a known calibration stress condition.

Problems experienced with oscilloscope display of stresses measured in the active input mode were caused by electromagnetic noise in the laboratory environs. Such problems are common to oscilloscope measurement of small signals and can be solved by use of adequate shielding.

The major recommendation of this report is that the passive, resistive state of stress analog computer be commercially developed as an inexpensive data reduction link to be used with existing, commercially available strain gage equipment.

In order to reduce the size of the required rectangular rosette to three elements, the author suggests that the computer design include a unity gain operational amplifier which would generate the fourth strain signal from the relation that $\varepsilon_{I} + \varepsilon_{III} = \varepsilon_{II} + \varepsilon_{IV}$. This approach would reduce the cost of the rosettes and of their installation. The disadvantage of this method is that no check on the validity of the three original signals would be available.

Production model computers should be constructed with precision resistors of tolerance 1% or less. Their use should improve the accuracy of the device without significantly affecting its price.

Further investigation of the Mohr's Circle resistance analog should be performed, particularly in the direction of a computing network which can solve the rosette problem using strain data from Delta Rosettes. Preliminary investigation of this topic indicates the Delta Rosette problem to be more difficult than the Four Gage Rectangular Rosette problem, because of the geometric limitations imposed by locating the nondimensionalized stress circle inside an equilateral triangle rather than inside a square.

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Appendix I . Detailed Development of the Computing Network Specific Resistance Relationships

The specific resistance relationships, Equations 10, were developed from the resistance-distance analog relationships, Equations 9, in the following manner.

From (9a)

$$2r_c = 2(2 + \sqrt{3})(1 - k)/k$$

obtain (10a)

$$r_c = (2 + \quad 3)(1 - k)/k.$$

Next look at (9b).

$$\frac{(2r_c + 1)(r_o + 1)}{2r_c + r_o + 1} = (2 - k\sqrt{3})/k$$

Let $a = (2 - k\sqrt{3})/k$; then regroup (9b) into the form

$$(2r_c + 1)a + ar_o = (2r_c + 1)r_o + (2r_c + 1)$$

and solve for r_o.

$$r_o = \frac{(a - 1)(2r_c + 1)}{2r_c + 1 - a}$$

Substitute the expressions for a and r_c into this expression for r_o and find that

$$r_o = \frac{8 + 4\sqrt{3} - (16 + 10\sqrt{3})k + (9 + 5\sqrt{3})k^2}{k[(2 + 2\sqrt{3}) - (3 + \sqrt{3})k]} ,$$

which simplifies to

$$r_o = \frac{(1 + \sqrt{3}) - (2 + \sqrt{3})k}{k} . \tag{10c}$$

Now look at (9c).

$$\frac{(2r_c + 1)r_o\,(r_m + 1)}{(2r_c r_o + r_o + 1)r_m} = \frac{(2 - k\sqrt{3})}{(\sqrt{3} - 1)k} = \frac{a}{\sqrt{3} - 1}$$

Rewriting the left hand expression in terms of (9b) yields

$$a\,\frac{r_o}{r_m}\,\frac{(r_m + 1)}{(r_o + 1)} = \frac{a}{\sqrt{3} - 1} .$$

Thus

$$\frac{r_m}{r_m + 1} = (\sqrt{3} - 1)\,\frac{r_o}{r_o + 1}$$

or

$$r_m = \frac{(\sqrt{3} - 1)\,r_o}{(2 - \sqrt{3})r_o + 1} .$$

Substitution of (10c) yields

$$r_m = \frac{2 - (1 + \sqrt{3})k}{\sqrt{3} - 1},$$

which may be put in the form

$$r_m = (1 + \sqrt{3}) - (2 + \sqrt{3})k. \tag{10b}$$

Appendix II. Prototype Resistance Values and Materials Costs

The potentiometer used for the resistance stress circle is a twelve tap, two brush circular potentiometer with a nominal resistance of 100,000 ohms before closing. The resistance between two adjacent taps, therefore, is 8333 ohms before closing. The specific resistances determined by choice of the design value and other selected values of Poisson's Ratio and by solution of Equations 10, 12, 13 and 14 are converted to actual resistances in ohms when multiplied by 8333.

The prototype computer design is based on the value $\mu = 0.28$ and has provision for one additional value, $\mu = 0.33$. The adjustment of Poisson's Ratio is based on the use of 120 ohm strain gages and ballast resistors.

The calculated specific resistances and the calculated resistances as well as the nominal resistance values actually used in prototype construction are tabulated below. One half watt, five percent resistors were used throughout.

For $\mu = 0.28$, $k = 0.562$.

Symbol of Specific Resistance	Calculation Specific Value	Calculation Resistance (ohms)	Resistance Used (ohms)
r_{ch}	1.873	15,600	15,560
r_m	0.638	5317	5320
r_o	1.137	9460	9400
kr_h	6.740	56,200	56,000
r_μ	1.018	848	820

All of the nominal resistance values used in prototype construction are well within the five percent tolerance of the resistors.

The cost of materials used in construction of the prototype computer is tabulated below. Prices are those in effect in January, 1969.

Quantity	Item Description	Total Cost
1	Potentiometer (12 taps - 2 brushes)	$100.00
1	Chassis (12 x 7 x 2")	1.65
1	Vector Board	2.68
4	Insulators	0.92
1	Switch (4 section-12 position)	3.42
9	Terminals (multi-connection)	2.88
40	Resistors (1/2 watt-5%)	2.80
1	Dial Plate	0.42
2	Knobs	0.89
4	Rubber Feet	0.44
Misc.	Hardware	0.36
	Total Cost	$116.46

A rough estimate of production costs places the customer price of a production model of the computer at about $150. The price was estimated as follows:

Potentiometer cost (in lots of 25)	$25.00
Other materials	50.00
Four hours of labor (including overhead) at $10.00 per hour	40.00
Thirty percent markup	35.00
Total Price	$150.00